High School Mathematics Competition

Final Round, February 4, 2011

1. An equilateral triangle is inscribed in a circle. A line parallel to a side of the triangle divides the triangle in two parts of equal area. Compute the ratio of the areas of the circle inscribed in the smaller triangle and the original circle.

2. Find all integers x and y satisfying the inequality

$$x^4 - 12x^2 + x^2y^2 + 30 \leq 0.$$

3. The points D and E divide the basis BC of the isosceles triangle ABC in three equal parts, and D is between B and E. Show that $\measuredangle BAD < \measuredangle DAE$.

4. Show that there exists a square number (i.e. a square of a positive integer) such that the sum of its digits equals 2011.

5. Two players, *Builder* and *Breaker* play the following game. Builder starts. Both players pick in turn elements from the set $\{0, 1, \ldots, 10\}$. Builder wins, if some four of the six numbers she picks form an arithmetic sequence. Breaker wins, if he can prevent Builder from forming such an arithmetic foursome. Which of the players has a winning strategy?

Time allowed: **3 hours**.

Each answer on a different sheet of paper.

Write your name and contact information (school, home address and email address) clearly on the paper.