31. 10.

High School Mathematics Competition

First Round, Basic Level

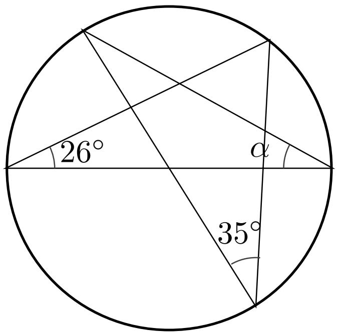
2016

The problems are on two pages; the first six problems are multiple choice problems with zero to four correct answers. **The use of calculators is not allowed.**

1. A merchant bought a quantity of unroasted coffee, roasted it and then sold it for a euros per kilogram. In roasting, the coffee loses 20 % of its weight. The merchant's profit is 20 %. The buying price (€/kg) was smaller than the selling price (to one percentage point) by

a) 20 %	b) 25 %	c) 33 %	d) 40 %

2. In the picture, some inscribed angles have been drawn.



Deduce the size of angle α.

a) 23°	b) 29°	c) 34°	d) 35°

3. Consider the expressions $A = (a^2 + b^2)(c^2 + d^2)$ and $B = (ac + bd)^2$. Then

a) $A > B$ for all real numbers a, b, c and d.
b) $A \geq B$ for all real numbers a, b, c and d.
c) $A > B$ for $a = 12$, $b = 5$, $c = 8$ and $d = 3$.
d) There are real numbers a, b, c and d such that $A = B$.

4. The perimeter of a circular sector is 40 cm, and its area is maximal among sectors of this kind. Then:

a) The arc of the sector and the radius of the circle are of equal length.
b) The arc of the sector is twice as long as the radius.
c) The radius of the circle is 10 cm.
d) The central arc of the sector is 90°.

5. Let

$$P = \left(1 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{2015}\right)\left(1 + \frac{1}{2016}\right).$$

Then

a) P is an integer b) $P > 1000$ c) $P < 2016$ d) $P = 3110$

6. We write the polynomial $P(x) = (2x+1)^5$ in the expanded form $P(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$. Which of the claims on the coefficients of the polynomial $P(x)$ are true?

a) The sum $a_5 + a_4 + a_3 + a_2 + a_1 + a_0$ is a multiple of three.
b) At least one of the coefficients (a_0, a_1, ... or a_5) is a multiple of three.
c) The sum of the coefficients is a multiple of five.
d) At least one of the coefficients is a multiple of five.

7. The triangle ABC is isosceles, and $\sphericalangle BAC > 30°$. D is a point on the base BC and E is a point on the leg AC. We assume that $\sphericalangle BAD = 30°$ and $|AD| = |AE|$. Determine $\sphericalangle EDC$.

8. Solve the equation

$$\sqrt{2 + 4x - 2x^2} + \sqrt{6 + 6x - 3x^2} = x^2 - 2x + 6.$$

Basic Level Multiple Choice Answer Sheet

31. 10. 2016

The first six problems are multiple choice problems. Their answers should be written in the table below. Each multiple choice problem has 0 to 4 correct answers. Put a "+" to the appropriate square, if the answer is right and a "−" if the answer is wrong. All correct marks give one point and incorrect or unintelligible marks give zero points. The answers to problems 7 and 8 can be written on a separate paper. For each of these problems, a maximum of 6 points is given.

The time allowed is 120 minutes. Please write your name and school with block letters on every paper you return.

Name : ______________________________

School : ______________________________

Home address : ______________________________

Email : ______________________________

	a	b	c	d
1.				
2.				
3.				
4.				
5.				
6.				

31. 10.

High School Mathematics Competition

First Round, Intermediate Level

2016

1. The perimeter of a circular sector is 40 cm, and its area is maximal among sectors of this kind. Then:

a) The arc of the sector and the radius of the circle are of equal length.
b) The arc of the sector is twice as long as the radius.
c) The radius of the circle is 10 cm.
d) The central arc of the sector is 90°.

2. For which values of the constant $a \in \mathbb{Z}$ is the polynomial $P(x) = x^{2016} - x^{1000} + 800x^{15} + ax^7 - 2$ divisible by the polynomial $Q(x) = x^{312} - 41x^{192} + 5x^8 - x$?

a) $a = -1$, b) $a = 1$ c) all $a \in \mathbb{Z}$ d) no $a \in \mathbb{Z}$

3. What can be said of the integer solutions of the Diophantine equation

$$x^2 + 5y^4 = 2016?$$

a) Such solutions exist.
b) All solutions satisfy $|x| < 50$ and $|y| < 5$.
c) $x + 1$ or $x - 1$ is a multiple of 5.
d) $x \neq 0$ and $y \neq 0$.

4. The triangle ABC is isosceles, and $\sphericalangle BAC > 30°$. D is a point on the base BC and E is a point on the leg AC. We assume that $\sphericalangle BAD = 30°$ and $|AD| = |AE|$. Determine $\sphericalangle EDC$.

5. Six couples will be divided into groups in such a way that no group contains both members of any of the couples.

a) In how many ways can they be divided into two groups of six persons each?
b) What about groups of four persons?

6. Find all positive integers x and y such that the number $x^4 + 4y^4$ is a prime number.

31. 10.

Intermediate Level Multiple Choic
Answer Sheet

2016

The first three problems are multiple choice problems. Their answers should be written in the table below. Each multiple choice problem has 0 to 4 correct answers. Put a "+" to the appropriate square, if the answer is right and a "−" if the answer is wrong. All correct marks give one point and incorrect or unintelligible marks give zero points. The answers to problems 4 to 6 can be written on a separate paper. For each of these problems, a maximum of 6 points is given.

The time allowed is 120 minutes. Please write your name and school with block letters on every paper you return.

Name : ________________________________

School : ________________________________

Home address : ________________________________

Email : ________________________________

	a	b	c	d
1.				
2.				
3.				
4.				
5.				
6.				

31. 10.

High School Mathematics Competition
First Round, Open Level

2016

1. Determine the smallest positive integer k such that the number $10!/k$ is a perfect square, i.e., the second power of some integer m. Find this integer m. (For a positive integer n, the *factorial* of n is the product $1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$ and is denoted by $n!$.)

2. The diameter of a circle is AB, and the tangent at A and a chord BC are drawn. The extension of BC meets the tangent at D. Show that the tangent to the circle at C intersects the segment AD at its midpoint.

3. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the inequality

$$\frac{f(xy) + f(xz)}{2} - f(x)f(yz) \geq \frac{1}{4}$$

for x, y, $z \in \mathbb{R}$.

4. Aino and Väinö play the game GCD(m, n), where m and n are positive integers. At the start of the game there are two heaps of stones on the table, one containing m and the other one n stones. At her/his turn a player can remove from one of the heaps any positive multiple of the number of stones in the other heap. The players remove stones in turn, and Aino starts. The player who is able to take all stones from one heap wins. Show that there exists an $\alpha > 1$ such that whenever m, $n \in \mathbb{N}$ and $m \geq \alpha n > 0$, Aino has a winning strategy in GCD(m, n), i.e. whatever Väinö's moves are, she can secure victory to her.