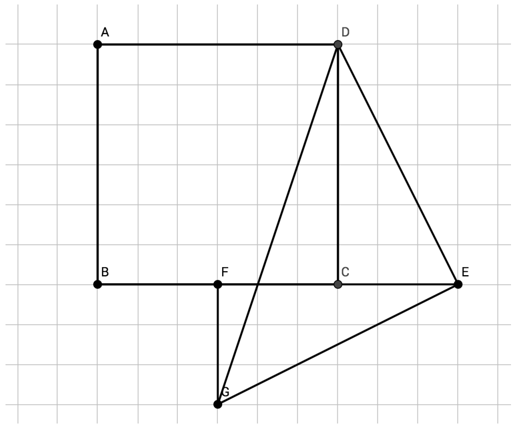
Competition finals for 7:th graders of Helsinki, 2018

1. In this problem, and only this problem, it is enough to give just the answer without any justifications or intermediate steps.

a) Compute $\frac{1}{2} - \frac{1}{32} + \frac{7}{64}$. (Please give the answer in fractional form.)

b) Pictured is a grid with a square $ABCD$, congruent triangles CED and FEG, and a triangle DEG. The grid consists of equally large squares with sides of length 1. What is the area of the triangle DEG?



2. How many of the numbers

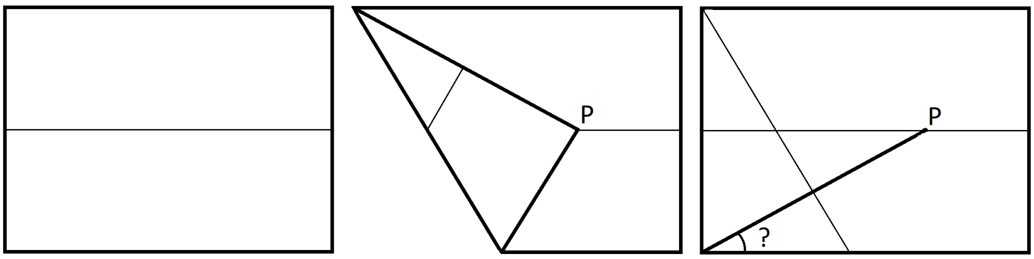
$$\frac{101}{1}, \frac{102}{2}, \frac{103}{3}, \ldots, \frac{200}{100}$$

are integers (the numbers are of the form $\frac{k+100}{k}$ where k is an integer and $1 \leq k \leq 100$)?

3. Is the number $1^{2018}+2^{2018}+5^{2018}$ divisible by 10? [Here a^{2018} stands for the product $a \cdot a \cdot \ldots \cdot a$ of 2018 copies of the number a.]

4. Pictured is a rectangular sheet of paper with a horizontal crease halfway up the paper. The sheet is folded so that the fold starts from the upper left corner and takes the lower left corner onto the horizontal crease. Let P be the point the lower left corner lands on in this fold.

If we open the paper and draw a line from the lower left corner to P, then how large is the angle between the line and the bottom edge of the paper?



5. A triangle is formed out of positive integers (which are greater than or equal to 1) in the following manner: the numbers are assembled into floors, every floor has one less integer than the floor below it, and starting from the second floor, every integer is the sum of the two below it. We call such a construction a *number triangle*. Below is an example of a four floors tall number triangle with the number 30 at the top.

30

18 12

11 7 5

6 5 2 3

How many different five floors tall number triangles are there with the number 17 at the top?